Wavelet-based decomposition and analysis of structural patterns in astronomical images

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ABSTRACT

Context. Images of spatially resolved astrophysical objects contain a wealth of morphological and dynamical information, and effective extraction of this information is of paramount importance for understanding the physics and evolution of these objects. Algorithms and methods employed presently for this purpose (such as, for instance, Gaussian model fitting) often use simplified approaches for describing the structure of resolved objects.

Aims. Automated (unsupervised) methods for structure decomposition and tracking of structural patterns are needed for this purpose, in order to be able to deal with the complexity of structure and large amounts of data involved.


Results. The method is tested against simulated images of relativistic jets and applied to data from long-term monitoring of parsec-scale radio jets in 3C 273 and 3C 120. Working at its coarsest resolution, WISE reproduces exceptionally well the previous results of model fitting evaluation of the structure and kinematics in these jets. Extending the WISE structure analysis to fine scales provides the first robust measurements of two-dimensional velocity fields in these jets and indicates that the velocity fields are likely to reflect the evolution of Kelvin-Helmholtz instabilities developing in the flow.

Key words. methods: data analysis – galaxies: jets – galaxies: individual: 3C 120 – quasars: individual: 3C 273

1. Introduction

Steady improvements of dynamic range of astronomical images and ever increasing complexity and detail of astrophysical modeling bring a higher demand on automatic (or unsupervised) methods for characterisation and analysis of structural patterns in astronomical images.

A number of approaches developed in the fields of computer vision and remote sensing show promise for tracking structural changes (cf., Yuan et al. 1998; Doucet & Gordon 1999; Arulampalam et al. 2002; Sidelnikblad et al. 2004; Doucet & Wang 2005; Myint et al. 2008) either require oversampling in the temporal domain or rely on multiband (multicolor) information underlying the changing patterns. This renders them difficult to be used in astronomical applications that typically focus on tracking changes in brightness in a single observing band, monitored with sparse sampling, with structural displacements between individual images frames often exceeding the dimensions of the instrumental point spread function (PSF).

Astronomical images and high-resolution interferometric images in particular offer very limited (if any) opportunity to identify “ground control points” or to build “scene sets” as employed routinely in remote sensing and machine vision applications (cf., Damjdi et al. 1997; Zheng & Chellappa 1999; Adams & Williams 2003; Zitová & Flusser 2003; Paulson et al. 2010). Structural patterns observed in astronomical images often do not have a defined or even preferred shape, which is an aspect relied upon in a number of the existing object recognition algorithms, (e.g., Agarwal et al. 2003). Astronomical objects normally do not feature sufficiently robust edges warranting application of edge-based detection and classification commonly used in object recognition methods (Belongie et al. 2002). In addition to this, astronomical images often deal with partially transparent, optically thin structures in which multiple structural patterns can overlap without full obscuration, which makes such images even more difficult to be analyzed using the algorithms developed for the purposes of remote sensing and computer vision. Because of these specifics, automated analysis and tracking of structural evolution in astronomical images remains very challenging, and it requires an implementation of a specially designed approach that can deal with all of the main specific characteristics of astronomical imaging of evolving structures.

Presently, structural decomposition of astronomical images normally involves simplified supervised techniques based on identification of specific features of the structure (e.g., ridge lines, Hummel et al. 1992; Lobanov 1998b; Bach et al. 2008), analysis of image brightness profiles (cf., Lobanov & Zensus 2001; Lobanov et al. 2003) or fitting the observed structure with a set of predefined templates (e.g., two-dimensional Gaussian features). Two-dimensional cross-correlation has been attempted only in a very limited number of cases (e.g., Biretta et al. 1995; Walker et al. 2008), each time requiring manual segmentation of images, which had imposed strong limitations on the number of structural patterns that could be tracked.

In some particular situations, such as for instance in images of extragalactic radio jets, distinct structural patterns cover a variety of scales and shapes, e.g., from marginally resolved brightness enhancements due to relativistic shocks embedded in the flow (Zensus et al. 1995; Unwin et al. 1997; Lobanov & Zensus 1999) to thread-like patterns produced by plasma instability (Lobanov 1998b; Lobanov & Zensus 2001; Hardie et al. 2005). In the course of their evolution, most of these patterns may ro-

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tate, expand, deform, or even break up into independent substructures. This makes template fitting and correlation analysis particularly challenging, and simultaneous information extraction on multiple scales and flexible classification algorithms are required.

Deconvolution algorithms (cf., Høgbom 1974; Clark 1980), extended to multiple scales (e.g., Cornwell 2008), could in principle deal with this task. However, the comparison of structures imaged at different epochs is difficult, due to general non-uniqueness of the solutions provided by deconvolution and an obvious need to group parts of the solution together in order to describe structures that are substantially larger than the image PSF.

A more robust approach to automate identification and tracking of structural patterns in astronomical images can be provided by a generic multiscale method such as wavelet deconvolution or wavelet decomposition (cf., Starck & Murtagh 2006). While applied typically for image denoising and compactification, wavelets provide all ingredients necessary for decomposing the overall structure in an image into a robust set of statistically significant structural patterns. This paper explores the wavelet approach and presents a wavelet-based image segmentation and evaluation (WISE) method for structure decomposition and tracking in astronomical images. The method is based on combining wavelet decomposition with watershed segmentation and multiscale cross-correlation algorithms in order to deal with temporal sparsity of astronomical images, multiscale structural patterns, and their large displacements between individual image frames.

The conceptual foundations of the method are outlined in Sect. 2. An algorithm for segmented wavelet decomposition (SWD) of structure into a set of statistically significant patterns (SSP) is introduced in Sect. 3. A multiscale cross-correlation (MCC) algorithm for tracking positional displacements of individual SSP is described in Sect. 4. In Sect. 5 WISE is tested against simulated images of relativistic jets. In Sect. 6 applications of WISE to astronomical images of parsec-scale radio jets in 3C 273 and 3C 120 are described and compared with results of conventional structure analysis previously applied to these data. The results are discussed and summarized in Sect. 7.

2. Wavelet-based image structure evaluation (WISE) algorithm

2.1. The wavelet transform

The wavelet transform is a time-frequency transformation that decomposes a square-integrable function, \( f(x) \), by means of a set of analyzing functions, \( \psi_{a,b}(x) \), obtained by shifts and dilations of a spatially localized square-integrable wavelet function \( \psi(x) \), so that

\[
\psi_{a,b}(x) = \frac{1}{\sqrt{a}} \psi\left(\frac{x-b}{a}\right) \quad (a \neq 0),
\]

where \( a > 0 \) is the scale parameter and \( b \) is the position parameter. The Morlet-Grossman definition (A. Grossmann 1984) of the continuous wavelet transform for a one-dimensional function \( f(x) \in L^2(R) \), the space of all square-integrable functions, is:

\[
W(a,b) = \frac{1}{\sqrt{a}} \int f(x) \psi^*\left(\frac{x-b}{a}\right) dx.
\]

Different discrete realisations of the wavelet transform exist (Mallat 1989; Starck & Murtagh 2006). In the analysis presented here, the à trous wavelet (Holschneider et al. 1989; Shensa 1992) is used. The à trous wavelet transform has the advantage of yielding a stationary, isotropic, and shift-invariant transformation which is well-suited for astronomical data analysis applications (Starck & Murtagh 2006). Different scaling functions can be used with this transform (Unser 1999). The choice of the scaling function is guided by the specific properties of the image and the information required to be extracted from the image (Ahuja et al. 2005). In the following, we use the B-spline scaling function (also called the triangle function).

In this work, we consider digital astronomical images as sampled data \( c_0(k) \) defined as a scalar product (computed at locations \( k \) of a function \( f(x) \) (sky brightness distribution, convolved with the instrumental point-spread-function) with a scalar scaling function \( \phi(x) \), yielding

\[
c_0(k) = \langle f(x), \phi(x-k) \rangle.
\]

This operation corresponds to application of a low pass filter to a continuous function. The smoothed data \( c_j(k) \) at position \( k \) and a given resolution \( j \), containing information of \( f(x) \) on spatial scale \( > 2^j \) is given by

\[
c_j(k) = \frac{1}{2^j} \langle f(x), \phi\left(\frac{x-k}{2^j}\right) \rangle.
\]

The wavelet coefficients \( w_j(k) \), that contain information on spatial scales between \( 2^{j-1} \) and \( 2^j \), are then given by the difference between two consecutive scale resolutions:

\[
w_j(k) = c_{j-1}(k) - c_j(k).
\]

These expressions can be easily extended to a two-dimensional case. Applied to an image, it produces a set \( w_j(l,k) \) of resolution-related views of the image, which are called wavelet scales. The concept of spatial wavelet scale plays a role similar to that of a frequency: small scales correspond to high frequency and large scales to low frequency.

2.2. Conceptual structure of WISE

In order to characterize structure and structural evolution of an astronomical object, the imaged object structure needs to be decomposed into a set of significant structural patterns (SSP) which can be successfully tracked across a sequence of images. This is typically done by fitting the structure with predefined templates (such as two-dimensional Gaussians, disks, rings, or other shapes deemed suitable for representing particular structural patterns expected to be present in the imaged region; Fomalont 1999; Pearson 1999) and allowing their parameters to vary. It is clear, however, that for a robust structural decomposition made without a priori assumptions, also the generic shape of these patterns must be allowed to vary. To ensure this, a method is needed that can automatically identify arbitrarily shaped statistically significant structural patterns, quantify their significance, and provide robust thresholding based on the significance of individual features.

Multiscale decomposition provided by the wavelet transform (Mallat 1989) makes wavelets exceptionally well-suited to perform such a decomposition, yielding an accurate assessment of the noise variation across the image and warranting a robust representation of the characteristic structural patterns of the image. In order to further increase the robustness of the method, the multiscale approach is extended here to object detection, similarly to the methodology developed for the multiscale vision model (MVM; Rué & Bijaoui 1997; Starck & Murtagh 2006). Multiscale decompositions carried out in 3C 273 and 3C 120 are described and compared with results of conventional structure analysis previously applied to these data. The results are discussed and summarized in Sect. 7.

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Combining these features together, we have developed a new, wavelet-based image structure evaluation (WISE) algorithm aimed specifically at structural analysis of semi-transparent, optically thin structures in astronomical images. The method employs segmented wavelet decomposition (SWD) of individual images into arbitrary two-dimensional SSP (or image regions) and subsequent multiscale cross-correlation (MCC) of the resulting sets of SSP. A detailed description of the method is given below.

3. Segmented wavelet decomposition

Segmented wavelet decomposition (SWD) comprises the following steps for describing image structure by a set of statistically significant patterns:

1. A wavelet transform is performed on an image \( I \), decomposing the image into a set of \( J \) sub-bands (scales), \( w_j \), and estimating the residual image noise (variable across the image).

2. At each sub-band, statistically significant wavelet coefficients are extracted from the decomposition by thresholding them against the image noise.

3. The significant coefficients are examined for local maxima and a subset of the local maxima satisfying composite detection criteria is identified. This subset defines the locations of SSP in the image.

4. Two-dimensional boundaries of the SSP are defined with the image is segmented by watershed segmentation using the feature locations as initial markers. This step defines boundaries of two-dimensional regions associated with the individual SSP.

These steps essentially combine the MVM approach with watershed segmentation and a two-level thresholding for the purpose of yielding a robust SSP identification procedure that would improve the quality of subsequent tracking of SSP which have been cross-identified in a sequence of images of the same object.

The SWD decomposition delivers a set of scale-dependent models (SDM) each containing two-dimensional features identified at the respective scale of the wavelet decomposition. The combination of all SDM provides a structure representation that is sensitive to compact and marginally resolved features as well as to structural patterns much larger than the FWHM of the instrumental point-spread function (PSF) in the image. It should also be noted that individual SSP identified at different wavelet scales are partially independent, which allows for spatial overlaps between them and can be used for improving the robustness and reliability of detecting structural changes by cross-correlating multiple images of the same object.

3.1. Determination of significant wavelet coefficients

As has been discussed above, the wavelet transform of a signal produces a set of zero mean coefficient values \( w_j \) at each scale \( j \). To extract significant wavelet coefficients and filter out the noise, a threshold, \( \tau_j \), is determined by requiring \( |w_j| \geq \tau_j \) for the significant coefficients. The determination of \( \tau_j \) depends on the noise characteristics in the image and a false discovery rate (FDR) \( \epsilon \). Throughout the paper, it is assumed that the image noise is Gaussian. Techniques exists to handle other types of noise, for example using the Anscombe transform for Poisson noise. We refer to Stark & Murtagh (2006) for a complete review of noise treatment in wavelet analysis of images.

The wavelet transform does not change the Gaussian nature of the noise and hence the noise can be characterized at each scale of its wavelet decomposition by a zero mean and a standard deviation \( \sigma_j \). This property can be used for relating the desired noise threshold \( \tau_j \) by setting \( \tau_j = k_0 \sigma_j \) and requiring the significant coefficients to satisfy the condition \( |w_j(x,y)| \geq k_0 \sigma_j \). Choosing \( k_0 = 3 \) gives an FDR \( \epsilon = 0.002 \). The application of the threshold condition yields a denoised map for each wavelet scale:

\[
m_j(x,y) = \begin{cases} w_j(x,y) & \text{if } |w_j(x,y)| \geq k_0 \sigma_j \\ 0 & \text{otherwise} \end{cases}
\]

In order to determine \( \sigma_j \) from the standard deviation of the noise of the original image \( \sigma_s \), the standard deviations \( \sigma_j \) are calculated for each scale of the wavelet decomposition of simulated Gaussian-noise data with \( \sigma_{s, \text{sim}} \equiv 1 \). We then use the linearity of the wavelet transform to obtain \( \sigma_j \) from the relation \( \sigma_j = \sigma_s \sigma_j \) (Stark & Murtagh 1994).

An estimate of \( \sigma_s \) can be obtained using one of the several techniques available for this purpose. If a noise map can be accessed, \( \sigma_s \) is provided simply by calculating the standard deviation of the entire map or of the relevant areas in the map. In other situations, \( k \)-sigma clipping or Median Absolute Deviation (MAD) estimation (Stark & Murtagh 2006) can be applied to assess the noise properties in the image and obtain an estimate of \( \sigma_s \).

3.2. Localisation of significant structural patterns

A maximum filter is used to identify putative positions of SSP at each scale of the wavelet decomposition. The filter comprises applying the morphological operation of dilation with a structuring element of a desired size. The location of a local maxima occurs when the output of this operation is equal to the original data value. This defines a list of local maxima, \( H_j \), at the scale \( j \):

\[
H_j = \{ (x,y) : \text{dilation}(w_j(x,y)) = w_j(x,y) \}.
\]

The shape and size of the chosen structuring element have an impact on the minimal separation of two detected local maxima. For our specific application, we use a diamond structuring element of the size that matches the scale at which it is applied; with the minimum size of two pixels. Each of the lists \( H_j \) is clipped at a specific detection threshold, \( \rho_j \). This is done recalling that, for Gaussian noise, the detection level is proportional to \( \sigma_j \), hence \( \rho_j = k_d \sigma_j \) can be set. For successful detection thresholding, the condition \( k_d \geq k_0 \) must be satisfied (with \( k_d = 4-5 \) typically providing good thresholds).

The threshold clipping can be used for defining \( F_j \) as a group of significant feature locations:

\[
F_j = \{ f = (x,y) : (x,y) \in H \wedge |w_j(x,y)| \geq k_d \sigma_j \},
\]

and these locations can be used for the subsequent definition of SSP in the image.

3.2.1. Identification of significant structural patterns

An SSP is defined as a 2D region of enhanced intensity extracted at a given wavelet scale. To determine the extent and shape of individual SSP associated with significant local maxima, image

Murtagh (2006) and in related work on object and structure detection (Men'shchikov et al. 2012; Seymour & Widrow 2002).
The watershed flooding (Beucher & Meyer 1993) is used for that purpose. The borders between individual regions are determined from the common minima located between the adjacent regions. This is achieved by watershed flooding (Beucher & Meyer 1993). Fig. 1 illustrates application of the watershed segmentation in a one-dimensional case.

The watershed segmentation is performed on noised map $m_j$ at all scales $j$ with $F_j$ as “water sources” or markers. Each local maximum $f_a$ of $F_j$ gives a region $S_{ja}$ defined as

$$s_{ja}(x,y) = \begin{cases} m_j(x,y) & \text{if } (x,y) \text{ is inside the watershed line of } f_a \\ 0 & \text{otherwise} \end{cases}. \tag{9}$$

The resulting SSP representation of an image at the scale $j$ is finally derived as the group of regions:

$$S_j = \{ s_{ja} : f_a \in F_j \}. \tag{10}$$

An example of application of the SSP identification is shown in Figures 4 for a simulated image of a compact radio jet.

4. Multiscale cross-correlation

In order to detect structural differences between two images of an astronomical object made at epochs $t_1$ and $t_2$, one needs to find an optimal set of displacements of the original SSP (described by the groups of SSP $S_{j1}, j = 1,..., J$ that would match the SSP in the second image (described by $S_{j2}, j = 1,..., J$). Cross-correlation of the $S_{j1}$ and $S_{j2}$ is a natural tool for this purpose. There are however two specific issues that should be addressed, in order to ensure robustness of the cross-correlation analysis. Firstly, a viable rule should be introduced for identifying the relevant image area over which the cross-correlation should be applied. The typical choices of using the full image area or selecting manually the relevant fraction of the image (cf., Pushkarev et al. 2012, Fromm et al. 2013) are not satisfactory for this purpose. Secondly, the probability of false matching should be minimized for features with sizes smaller than the typical displacement occurred between the two epochs.

These two issues can be resolved by multiscale cross-correlation (MCC) combining together the structural and positional information contained in $S_j$ at all scales of the wavelet decomposition. The MCC uses a coarse-to-fine hierarchical strategy well known in the area of image registration. This principle has been first used in Vanderbrug & Rosenfield (1977) and Witkin et al. (1987) using Gaussian pyramids and then extended to the wavelet transform by Djamdji et al. (1993) and Zheng & Chellappa (1993). We refer to Zitova & Flusser (2003) and Paulson et al. (2010) for a review on the different techniques developed in this area. However, none of these algorithms can be directly applied for our purpose. The main reasons for this difficulty are the following:

1. The images we consider are sparsely sampled (with structural displacements of the order of the PSF size or even larger) and do not offer a set of “ground control points” facilitating image registration (while this aspect is a critical feature of virtually all of the remote sensing and computer vision algorithms).
2. The images are often dominated by optically thin structures (with the possibility of two or more independent structural features projected onto each other and often having different displacement/velocity vectors).
3. The structural patterns do not have a defined or even preferred shape, and their shape may also vary from one image to another.

All these aspects call for a method which differs significantly from the approaches used in the fields of remote sensing and computer vision.

Considering that SWD SSP at the wavelet scale $j$ have a typical size of $2^j$, the maximum displacement detectable on the scale $j$ must be smaller than $2^j$. Identification of the structural displacements can then be started from choosing $J$, the largest scale of the wavelet decomposition, such that it exceeds the maximum expected displacement but still satisfies the upper limit on $J$ given by the largest scale containing statistically significant wavelet coefficients. After correlating $S_{j1}$ with $S_{j2}$, the respective correlations between $S_{j1}$ and $S_{j2}$ on smaller scales are restricted to within the areas covered by $S_{j1}$ and $S_{j2}$ respectively, in the two images. Alternatively, this approach can also be used iteratively, restricting correlations on a given scale $j$ to within the areas of the correlated features identified at the $j+1$ scale. This algorithm is illustrated in Fig. 2. Details of the procedure for relating SSP identified at different scales are discussed in the next section.

4.1. Multiscale relations

Multiscale relations between SSP identified at different spatial scales can be derived from the basic region properties. Let us note again that sizes of SSP identified at the scale $j$ are of the order of $2^j$. Hence, any two individual SSP $S_a$ and $S_b$ of $S_j$, identified around respective local maxima $f_a$ and $f_b$, are separated from each other by at least $2^j$. This corresponds to the

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1 The watershed flooding earns its name from effectively corresponding to placing a “water source” in each local minimum and “flooding” the image relief from each of these “sources” with the same speed. The moment that the floods filling two distinct catchment basins start to merge, a dam is erected in order to prevent mixing of the floods. The union of all dams constitutes the watershed line.
The correlation is calculated between a reference image \( r \) and a target image \( t \), with the time order of the two images not playing any role. The correlation coefficients can be estimated using a number of different correlation criteria (see Giachetti (2000) for a review). The most commonly used criteria are the cross correlation,

\[
C_{CC}(r,t) = \sum r_i t_i ,
\]

and the sum of squared differences,

\[
C_{SSD}(r,t) = \sum (t_i - r_i)^2 .
\]

with \( i \) the pixel index. The tolerance to an offset between the reference and the target image is obtained by subtracting the mean value of the image intensity (zero-mean correlation). Similarly, tolerance to scale change is obtained by dividing the root-mean-square of the image intensity (normalized correlation).

The MCC algorithm is required to be insensitive to both the image offset and scale change. The zero-mean normalized cross correlation (ZNCC) and zero-mean normalized sum of squared difference (ZNSSD) can be applied for this purpose. It has been demonstrated in Pan et al. (2010) that these two criteria are equivalent. MCC uses the ZNCC method, based on its excellent computational performance (Lewis 1995). The ZNCC is given by

\[
C_{ZNCC}(r,t) = \frac{\sum \overline{r_i} t_i}{\sqrt{\sum \overline{r_i}^2 \sum t_i^2}} .
\]

with \( \overline{r} = r - \overline{r} \), and \( \overline{r} \) being the mean of \( r \). This criterion reaches maximum value of unity, when the reference and target image are identical.

In order to detect structural changes between the reference and target images, each single SSP \( s_{jj+1} \) of the reference image is cross correlated with the target image. As every SSP is constrained to be located a specific region, one is actually only interested in determining the correlation over that region. In order to achieve this, a weighting function, \( \omega \) is introduced which is normalized to unity and provides \( \omega \equiv 0 \) everywhere except inside the region containing the SSP of interest. A weighted zero-mean normalized cross correlation (WZNCC) can then be defined as

\[
C_{WZNCC}(r,t) = \frac{\sum \omega_i r_i t_i}{\sqrt{\sum \omega_i^2 \sum t_i^2}} .
\]

4.3. Detection of SSP displacements

As shown in Sect. 4.1, the displacements of individual features are determined starting from the largest scale and progressing to the finest scale of the wavelet decomposition. For each SSP at the scale \( j \), an initial guess for its displacement is provided by the displacement measured for the region at the scale \( j+1 \) which includes the SSP in question. The initial guess is then refined via the cross correlation.

This simple procedure is complicated by the fact that individual SSP may merge, split, or overlap as a result of structural changes occurring between the two observations. As a result, the displacement for which cross correlation is maximized does not necessarily provide the correct solution. Such a situation is exemplified in Fig. 3. In this example, SSP \( b \) is moving faster than SSP \( a \). As a consequence, cross correlation of the SSP \( b \) at the epoch \( t_i \) with \( w_{j2} \) yields the global maximum at \( x_{i0}^b \) and a local maximum at \( x_{i1}^b \). The formal cross correlation solution will be in error in this case. In order to avoid such errors (or at least to reduce their probability), it is necessary to cross identify groups of close SSP that can be related (i.e., causally connected) to each
The result of WZNCC between SSP panel and target image is plotted, with two detected SSP marked with Schematic illustration of the detection method used for two panels the wavelet decomposition at a scale $j$ of the reference (top panel) and target image is plotted, with two detected SSP marked with colors and letters. The x-axis of both panel is given in pixels and the y-axes indicate amplitude of the wavelet reconstruction on the scale $j$. The result of WZNCC between SSP $b$ and the target image is plotted below in the third panel. Two potential displacements are identified within the bounds (gray area) defined by Eq. (13) and the initial displacement guess $\Delta^{j+1}$ obtained from analysis at scale $j + 1$. In order to select the correct one, and to reduce the chance for erroneous cross-correlation, the group motion of causally connected SSPs (in this case SSP a and SSP b) is also included in the cross-correlation analysis, which results in the identification of the displacement $\Delta_{j,b} = 12$.

other in the two images. The cross correlation can then be applied to these groups as well as to their individual members, so that a set of possible solutions is found for all SSP, and the final solution is determined through a minimization analysis applied to the entire group of SSP.

At the first step of this procedure, subsets of features $G_j$ are defined that are considered to be interrelated. As was discussed in Sect. 4.1 at the scale $j$, $f_0$ is independent from $f_0$ if $\|f_a - f_b\| > 2^{j+1}$. Then,

$$G_{j,a} = \{x_i, x_j \in F_j \land \forall x_j \in F_j \setminus G_{j,a}, \|x_i - x_j\| \geq 2^{j+1}\},$$

with

$$F_j = \sum_u G_{j,u}.$$  

At the second step, cross-correlation is applied, yielding several possible displacement vectors for each feature of such a group. Considering the multiscale relations described in Sect. 4.1, one can then calculate the correlation coefficients at $\delta = (\delta x, \delta y)$ for a given feature $f_0$ of a group $G_{j,a}$:

$$\gamma_{j,a}(\delta x, \delta y) = C_{WZNCC}(s_{j,a}(x + \Delta x_{j,a}^{j+1} + \delta x, y + \Delta y_{j,a}^{j+1} + \delta y), w_j^j(x, y),$$

with $\|\delta\| < 2^j$.

As illustrated by the example shown in Fig. 3 for complex and strongly evolving structures, it is possible that formally the best cross correlation solution provided by the largest $\gamma_{j,a,\max}$ may be spurious. Hence, in order to avoid such spurious estimates of the displacement vectors, one may want to consider all local maxima of $\gamma_{j,a}$ that are above a certain threshold $\kappa$ (with $\kappa$ usually set $\geq 0.8$) as possible relevant solutions. These local maxima are found using the maximum filter method described in Sect. 4.2.

After the identification of all relevant local maxima, the WZNCC of the group of features is calculated for each possible group solution, and the combination of individual displacement $\delta$ maximizing the group correlation is selected. This operation is repeated for all groups of features $G_{j,a}$. This approach provides a robust estimate of the statistically significant structural displacement vectors across the entire image and at each structural scale.

In summary, our cross correlation procedure comprises the following main steps:

1. Individual initial displacements and bounds are determined for each SSP using the relations Eq. (12) and Eq. (13).
2. Groups of causally connected features are defined.
3. Cross correlation analysis is performed using the WZNCC for the groups and each of their elements, resulting in a set of potential displacements.
4. The final SSP displacements are determined by selecting a combination of individual displacement that maximizes the overall group correlation.

4.4. Overlapping multiple displacement vectors

In images of optically thin structures, several physically disconnected regions with different sizes and velocities may overlap, causing additional difficulties for reliable determination of structural displacements (observations of transversely stratified jets would be one particular example of such a situation). Using the independence of SWD SSP recovered at different wavelet scales, the MCC method can partially recover such overlapping displacement components. The maximum detectable displacement inside a region is determined by the largest wavelet scale $j$ for which this region can be described by at least 2 SSPs. Then as described in Sect. 4.1, the maximum detectable displacement would be $2^j$. If velocity gradients, or multiple velocity components, are expected inside this region, then this might not be sufficient and you might want to start the analysis at a wavelet scale which describe your region by 3 or 4 different SSP.

The multiscale relations described in sect. 4.1 rely on the assumption that SSP detected at a scale $j$ move, on average, like their parent SSP detected at scale $j + 1$. This assumption sets limits for the detecting different speed at different scales. Between two scales $j + 1$ and $j$, this limit, determined by Eq. (12), is of the order of $2^j$. As of velocity difference approach this limit, matching became more difficult. If a very strong stratification or distinctly different overlapping velocity components are expected, it is possible to relax this constraint by introducing a tolerance factor $k_{tol}$, in Eq. (13):

$$\|\delta_{j,a}\| < k_{tol} \times 2^j.$$  

This modification may increase the formal probability of spurious matches, but the overall negative effect of introducing the tolerance factor will be largely moderated by the cross correlation part of the algorithm. A similar limit applies if the gradient of velocity inside a SSP is of the order of the SSP size.

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on the jet axis, at the position where simulated by a Gaussian cylinder with FWHM that vary in magnitude and direction. The underlying flow is similarity (often branded as “jet components”) moving with velocities smooth underlying flow pervaded by regions of enhanced brightness with a bright and compact narrow end (“base” of the jet) and structural displacements generated for a range of spatial scales. An example of a simulated image together with the SSPs used.

5. Testing the WISE algorithm

To test the application of the WISE algorithm, simulated images of optically thin relativistic jets are prepared, which contain divergent and overlapping velocity vectors manifested by structural displacements generated for a range of spatial scales.

The simulated jet has an overall quasi-conical morphology, with a bright and compact narrow end (“base” of the jet) and smooth underlying flow pervaded by regions of enhanced brightness (often branded as “jet components”) moving with velocities that vary in magnitude and direction. The underlying flow is simulated by a Gaussian cylinder with FWHM $w_{\text{jet}}$ evolving with the following relation:

$$w_{\text{jet}}(z) = r_0 \frac{z}{z_0 + z} + r_1 \frac{z}{z_1} \tan(\phi_0),$$

where $r_0$ is the width at the base of the jet, $z_0$ the axial $z$-coordinate of the jet base, and $z_1$ the $z$-coordinate of the point after which $w(z)$ increase linearly with an opening angle of $\phi_0$, and intensity $i_{\text{jet}}$ evolving with the relation:

$$i_{\text{jet}}(z) = i_0 \left( \frac{z}{z_0} \right)^\alpha,$$

where $\alpha$ is the damping factor.

The jet base is modeled by a Gaussian component located on the jet axis, at the position $z_0$. The moving features, also modeled by Gaussian components (with randomly distributed parameters), are added in the area defined by the jet after $z_1$. The resulting image is finally convolved with a circular or elliptical beam, in order to study the effect of different instrumental PSF on the WISE reconstruction of the simulated structural displacements. An example of a simulated image together with the SSPs detected with the SWD at 3 different scales is shown in Fig. 4.

5.1. Test results

To evaluate the performance of WISE, two sets of tests have been performed. The first set consists of testing the SWD algorithm for sensitivity to features at low SNR (sensitivity test) and for distinguishing close and overlapping patterns (separation test). At the second stage of testing the full WISE algorithm (combining the SWD and the MCC parts) is applied to evaluate the sensitivity of the method to detecting spatial displacements of individual patterns (displacement test). In the following discussion, we define the SNR of a feature as its peak intensity over the noise level in the image.

5.1.1. The sensitivity test

This test is designed to represent as closely as possible the generic use of the SWD algorithm for detecting and classifying structural patterns in astronomical images. The test is performed on a simulated image of a jet as illustrated in Sect. 5. For this particular simulation a circular PSF with a FWHM of 10 pixels is applied. Morphology of the underlying jet is given by the initial width $r_0 = 5$ FWHM and an opening angle of $8^\circ$.

Superimposed on the smooth underlying jet background, Gaussian features with different sizes and intensities are then added. The features are separated widely enough from each other to avoid overlapping. The SWD method is applied to the simulated image, and the SWD detections are then compare to the positions, sizes and intensities of the simulated features. For the purpose of comparison, we perform also a simple direct detection (DD) which consist of detecting local maximum which are above a certain threshold directly on the image. Similarily as for the SWD detection, the threshold for the DD method is set to $k_d \sigma_n$, where $k_d$ is the detection coefficient as defined in Sect. 3.2 and $\sigma_n$ the standard deviation of the noise in the image. When determining if a detected feature correspond to a simulated one, a tolerance of 0.2 FWHM of the beam size on the position is used.

The resulting fractional detection rates are shown in Fig. 6 for simulated features of three different sizes. One can see that the SWD method successfully recovers 95% of extended features at SNR $\geq 6$, which makes it a reliable tool for detecting the statistically significant structures in astronomical images. In this particular test the SWD method outperform the DD method by a factor of approximatively 4. As shown in Fig. 6 the detection limit is a function of the detection threshold $k_d$.

5.1.2. The separation tests

The separation tests are designed to characterize the ability of the SWD method to distinguish two close features. In this test, the images structure comprises two Gaussian components of finite size at which are partially overlapping. The two components are defined by their respective SNR, $S_1$ and $S_2$ and FWHM, $w_1$ and $w_2$, and they are separated by a distance $\Delta_z$. For the purpose of quantifying the test results, the fractional component separation $r_s = 2 \Delta_z/(w_1 + w_2)$ is introduced. The tests determine the smallest $r_s$ for various combinations of the component parameters which the two features are detected. The performance of the SWD algorithm is again compared with results from the application of the DD method introduced in Sect. 5.1.1.

In the first separation test, the ratio $\kappa = w_1/w_2$ is varied, while setting $S_1 = S_2 = 20$. Note that the features are partially overlapping at their half-maximum level for $r_s \leq \kappa/(1 + \kappa)$. The results of this test are shown in Fig. 7 with SWD always
performing better than the DD. In addition to this, the evolution of minimum detectable $r_s$ with $\kappa_w$ indicates two different regimes for SWD. For $1 \leq \kappa_w < 2$, SWD progressively outperforms DD, with the difference between the two getting larger as $\kappa_w$ increases. At $\kappa_w \geq 2$, SWD undergoes a fundamental transition, with both features ultimately being always detected (at the 2 pixel separation limit). This is the result of multiscale capability of SWD to identify and separate power concentrated on physically different scales.

In the second separation test, the ratio $\epsilon_s = S_1/S_2$ is varied for features with $w_1 = w_2 = 10$ pixels. The results of this test are shown in Fig. 8, with SWD performing progressively better than DD with increasing SNR ratio $\epsilon_s$.

Both tests show that SWD is successful at resolving out two close and partially overlapping features. Assuming that the simulated component width $w_2$ in both tests is similar to the instrumental PSF, one can interpret $r_s$ as $\approx 2/(1 + \kappa_w)$ PSF, implying that SWD successfully distinguishes two marginally resolved features separated by $\approx 0.35$ PSF $(1 + \epsilon_s)^{1/2}/(1 + \epsilon_s^2)^{1/2}$, which is close to the expected limit

$$ r_{s,\text{lim}} \approx \frac{2}{\sqrt{\pi}} \ln \left[ \frac{S_2(1 + \epsilon_s) + 1}{S_2(1 + \epsilon_s)} \right]^{1/2} \frac{(1 + \epsilon_s)^2}{\sqrt{1 + \epsilon_s^2}} \times \text{PSF} $$

Fig. 5. Fractional detection rates of the SWD method (blue lines) in comparison with the direct detection (DD, yellow lines), width: 0.2 (dashed line), 0.5 FWHM (dotted line) and 1 FWHM (plain line) of the beam. Limits above which at least 95 % of features are detected are 5.6, 6.6, 9.6 for the SWD method and 27.1, 28.1 and 34.6 for the DD method. The number of false detection at the above limit is 3% for the DD, while it stay null in the case of SWD.

Fig. 6. Fractional detection rates and false detection rate of the SWD method for a detection threshold $k_d = 2$ (dashed line), 3 (dotted line), 4 (dashed dotted line) and 5 (plain line).

Fig. 7. Characterization of the separability, $r_s$, of two close features with varying FWHM ratio, $\kappa_w$. Separation limit is determined for the SWD method (blue cross) and a direct detection method (yellow cross) as introduced in Sect. 5.1.1. The gray hatch area is the region in the plot for which the separation between the two features is below 2 pixels.

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Fig. 8. Characterization of the separability, $r_s$, of two close features with variable SNR ratio $\epsilon_s$. Separation limit is determined for the WISE method (blue cross) and a direct detection method (yellow cross) as introduced in Sect. 5.1.1. The gray hatch area is the region in the plot for which the separation between the two features is below 2 pixels.
5.1.3. The structural displacement test

These tests use the full WISE processing on a set of two simulated jet images, first using the SWD algorithm to identify SSP features in each of the images and then applying the MCC algorithm to cross-correlate the individual SSP and to track their displacements from one image to the other. The jet images are simulated using the procedure described in the beginning of this section. The total of 500 elliptical features are inserted randomly inside the underlying smooth jet, with their SNR spread uniformly from 2 to 20 and the FWHM of the features ranging uniformly from 0.2 to 1 beam size. The simulated structures are convolved with a circular Gaussian (acting as an instrumental PSF) with a FWHM of 10 pixels. A damping factor \( \alpha \) of -0.3 is used.

Positional displacements are introduced to the simulated features in the second image. The simulated displacements have both regular and stochastic (noise) components introduced as follows:

\[
\Delta x = f_x(x) + G_x, \quad \Delta y = f_y(x) + G_y,
\]

where \( f_x \) and \( f_y \) are the regular components of the displacement, and \( G_x \) and \( G_y \) are two random variables following the Gaussian distributions described by the respective means \( < G_x >, < G_y > \) and standard deviations \( \sigma_x, \sigma_y \). After the two images have been generated, SSP are detected independently in each of them with the SWD and subsequently cross-identified with the MCC.

The displacement test explores a kinematic scenario describing an accelerating axial outflow with a sinusoidal velocity component transverse to the main flow direction:

\[
f_x(x) = a + bx + cx^2, \quad f_y(x) = d \cos \left( \frac{2\pi x}{T} \right).
\]

Results of the WISE application are shown in Figs. 9–10 for \( a = -2, b = 0.02, c = 0.00012, d = 10, T = 200, \) for the stochastic displacement components with \( \sigma_x = \sigma_y = 2 \) and \( \sigma_x = \sigma_y = 5 \) (0.2 FWHM and 0.5 FWHM), respectively (all linear quantities are expressed in pixels). The maximum expected displacement between the two images is 40 pixels. The WISE analysis was performed on scales 2–6 (corresponding to 4–64 pixels).

The comparison between the simulated displacements and the displacements detected by WISE reveals excellent performance of the matching algorithm. To assess this performance we compute the root mean square of the discrepancies between the simulated and detected displacements:

\[
e_x = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\Delta x_i - f_x(x_i))^2}
\]

\[
e_y = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\Delta y_i - f_y(x_i))^2},
\]

where \( \Delta x_i, \Delta y_i \) are the measured \( x \) and \( y \) components of the displacement identified for the \( i \)th simulated component, and \( x_i \) is the position of that component along the \( x \) axis in the first simulated image. The \( e_x \) and \( e_y \) determined from the WISE decomposition do not exceed the \( \sigma_x \) and \( \sigma_y \) of the simulated data. For the first we obtain \( e_x = 0.19 \) and \( e_y = 0.20 \), while for the second test, we obtain \( e_x = 0.43 \) and \( e_y = 0.42 \). The number of positively matched features decreases with increasing stochastic component of the displacements, but the errors of WISE decomposition always remain within the bounds determined by the simulated noise.

These comparisons indicate that WISE performs very well even in the case of relatively large spurious and random structural changes (which may result from deconvolution errors, phase noise, and incompleteness of the Fourier domain coverage by the data). As one expects such spurious displacement at a level of \( \lesssim \) FWHM/\( \sqrt{\text{SNR}} \), WISE should be able to reliably identify displacement in regions detected at SNR \( \gtrsim 4 \).
6. Applications to astronomical images

We have tested the performance of WISE on astronomical images by applying it to several image sequences obtained as part of the MOJAVE long-term monitoring program of extragalactic jets with very long baseline interferometry (VLBI) observations (Lister et al. 2013 and references therein). The particular focus of the tests is on two prominent radio jets in the quasar 3C 273 and the Seyfert galaxy 3C 120. These jets show a rich structure, with a number of enhanced brightness regions inside a smooth and slowly expanding flow. This richness of structure on the one hand has always been difficult to be analyzed by means of fitting it by two dimensional Gaussian features, on the other hand it has always suggested that the transversely resolved flows may manifest a complex velocity field, with velocity gradients along and across the main flow direction (cf. Lobanov & Zensus 2001, Hardee et al. 2005).

The MOJAVE observations, with their typical resolution of 0.5 milliarcsecond (mas), resolve transversely the jets in both objects and, in addition, they also reveal apparent proper motions of 3 mas/year in 3C 120 (Lister et al. 2013), which makes these two jets excellent targets for attempting to determine the longitudinal and transverse velocity distribution.

The WISE analysis has been applied to the self-calibrated hybrid images provided at the data archive of the MOJAVE survey. The results of WISE algorithm are compared to the MOJAVE kinematic modelling of the jets based on the Gaussian model fitting of the source structure (see Lister et al. 2013 for a detailed description of the kinematic modelling).

6.1. Analysis of the images

For each object, the MOJAVE VLBI images have been first segmented using the SWD algorithm, with each image analysed independently. The image noise have been estimated by computing \( \sigma_j \) at each wavelet scale, as described in Sect. 3.1. Based on these estimates, a \( \sigma_j \) thresholding has been applied subsequently at each scale. This procedure provides a better account for the scale dependence of the noise in VLBI images (Lal et al. 2010, Lobanov 2012), which is expected to result from a number of factors including the coverage of the Fourier domain and deconvolution.

Following the segmentation of individual images, MCC has been performed on each consecutive pair of images, providing the displacement vectors for all SSP that have been successfully cross-matched. The images have been aligned at the position of the SSP which is considered to be the jet “core” (which is typically, but not always, the brightest region in the jet). This is done in order to account for possible positional shifts resulting from self-calibration of interferometric phases and for potential positional shifts (core shift) due the opacity at the observed location of the jet base (Lobanov 1999b, Kovalev et al. 2008).

For SSP that have been cross-identified over a number of observing epochs, the combination of these displacements have provided a two-dimensional track inside the jet. The track information from several scales has also been combined, whenever a given SSP could be cross-identified over several spatial scales.

6.1.1. Jet kinematics in 3C 273

The MOJAVE database contains 69 images of 3C 273, with the observations covering the time range from 1996 to 2010 and providing, on average, one observation every three months. The SWD has been performed with four scales, ranging from 0.2 mas (scale 1) to 1.6 mas (scale 4).

For the MCC part of WISE, the individual images have been aligned at the positions of their respective strongest and most compact components (“core” components) as identified by the MOJAVE model fits. The kinematic evolution of most of the detected SSP is fully represented by the MCC results obtained for a single selected SWD scale. However, long-lived features in the flow could eventually expand so much that the wavelet power associated with a specific SSP would be shifted to a larger scale and the full evolution of such a feature has been described by a combination of MCC applications to two or more SWD scales.

The core separations of individual SSP obtained from WISE decomposition are compared in Fig. 11 to the results from the MOJAVE kinematic analysis based on the Gaussian model fitting of the jet structure. To provide this comparison, the effective resolution of WISE must be reduced by excluding the scales 1–2 from the consideration. Comparison of the MOJAVE and WISE results in Fig. 11 indicates that WISE detects consistently nearly all the components identified by the MOJAVE model fitting analysis, with a very good agreement on their positional locations and separation speeds.

The two dimensional tracks of the WISE features detected with this procedure are shown in Fig. 12 overplotted on a single-epoch image of the jet. The displacement tracks show clearly the presence of several “flow lines” threading the jet, which can be associated with the instability pattern identified in it (Lobanov & Zensus 2001). Some of these tracks can also be identified in Gaussian model fitting, but only if there is no substantial structural variations across the jet. If this is not the case, Gaussian model fitting becomes too expensive and too unreliable for the purpose of representing the structure of a flow. In such a situation, WISE provides a better way to deal with the structural complexity. We can conclude therefore that WISE can be applied for the task of automated structural analysis of VLBI images of jets (and similar sequences of images of objects with evolving structure), yielding a great increase of the speed of the analysis (it should be noted that the analysis of 69 images of 3C 273 took about 10 minutes of computing, while the model fitting of these images required a number of days of researchers’ time).

However WISE can certainly go beyond the resolution of Gaussian model fitting, by including also the scales which are smaller than the transverse dimension of the flow. An example of such an improvement is shown in Fig. 13 which focuses on MOJAVE observations of 3C 273 made between November 2003 and December 2006. At core separations larger than about 2 mas, WISE persistently detects several features at locations where the Gaussian model fits have been restricted to representing the structure with a single component. This is a clear sign of transverse structure in the flow, which is illustrated well by the respective displacement tracks shown in Fig. 13. These tracks provide strong evidence for a remarkable transverse structure of the flow, with three distinct flow lines clearly present inside the jet. These flow lines evolve in a regular fashion, suggesting a pattern that may rise as a result of Kelvin-Helmholtz instability, possibly due to one of the body modes that have been previously identified in the jet based on a morphological analysis of the transverse structure (Lobanov & Zensus 2001). That analysis also implied that the flow pattern should rotate counterclockwise, and this rotation is consistent with the general southward bending of the displacement vectors (particularly visible in Fig. 4 at distances of 4.5–6 mas).

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Fig. 11. Core separation plot of the most prominent features in the jet of 3C 273. The model-fit based MOJAVE results (dashed lines) are compared to the WISE results (solid lines) obtained for the SWD scales 3 and 4 (selected in order to match the effective resolution of WISE to that of the Gaussian model fitting employed in the MOJAVE analysis). A detailed analysis, also including the SWD scales 1 and 2, has been performed for the observations made between 11/2003 and 12/2006 (gray box), and its results are shown in Fig. 13.

Fig. 12. Two-dimensional tracks of the SSP detected by WISE at scales 3–4 of the SWD and compared in Fig. 13 to the features identified in the MOJAVE analysis of the images. The tracks are overplotted on a stacked-epoch image of the jet rotated by an angle of 0.55 radian. Colors distinguish individual SSP continuously tracked over certain period of time. Several generic “flow lines” clearly visible in the jet. These patterns are difficult to detect with the standard Gaussian model fitting analysis. The image is rotated.

6.1.2. 3C120

The MOJAVE database for 3C 120 comprises 87 images from observations made in 1996–2010, averaging to one observation every 3 month (but with individual gaps as large as one year). We prepare these images for WISE analysis, using the same approach as has been applied for 3C 273. In order to ensure sensitivity to the expected displacements of ≤ 3 mas between subsequent images, the application of SWD has been performed on five scales, from 0.2 mas (scale 1) to 3.2 mas (scale 5).

Applied to the MOJAVE images of 3C 120, WISE detects a total of 30 moving SSP. The evolution of 24 SSP is fully traced at the SWD scale 2 (0.4 FWHM), and combining two SWD scales is required to describe the evolution of the six remaining SSP. The resulting core separations of the SSP plotted in Fig. 15 are generally in a very good agreement with the separations of jet components identified in the MOJAVE Gaussian model fit analysis. For the moving features, displacements as large as ~ 3 mas have been reliably identified during the periods with the least frequent observations.
Separation from core (mas)
□1.5
0.0
1.5
Relative DEC (mas)
□4.5□3.0□1.50.01.53.0
Relative RA (mas)
11/03 06/04 12/04 05/05 09/05 03/06 06/06 09/06 12/06

Fig. 14. Two-dimensional tracks of SSP detected in 3C 273 at the scale 2 of SWD, for the epochs between 11/2003 and 12/2006. The tracks correspond to the features plotted in Fig. 13. Colors of the displacement vectors indicate epoch of measurement as shown in the wedge at the top of the plot. The plot confirms the presence of significant transverse structure in the jet, with up to three distinct flow lines showing strong and correlated evolution. Apparent inward motion detected in a nuclear region (0–0.3 mas) is most likely an artifact of a flare in the jet core.

Fig. 13. Core separation plot of features detected in a detailed analysis of the jet of 3C 273 which includes the SWD scales 1 and 2. Dashed lines show the MOJAVE model fit components, colored tracks present the SSP detected and tracked by WISE. At core separations ≳ 2 mas, WISE detects a larger number of significant features, as the jet gets progressively more resolved in the transverse direction (indicating also that structural description of the jet provided by Gaussian model fitting is no longer optimal).

The only obvious discrepancy between the two methods are the quasi-stationary features identified in the MOJAVE analysis, but not present in the WISE results. A closer inspection of the wavelet coefficients recovered at the SWD scale 1 also does not yield a statistically significant detection of an SSP at the location of the MOJAVE stationary component.

It should be noted that the stationary feature identified in the MOJAVE analysis is often separated by less than 1 FWHM from the bright core, while being substantially (factors of ~ 50–100 weaker than the core. Such an extreme flux density ratio between two clearly overlapping components may cause difficulties for the weaker feature to be identified against the formal thresholding criteria of WISE. The fact that the Gaussian model fitting has been performed in the Fourier domain (not affected by convolution) may have given it an advantage in such a particular setting. Subjective decision making during the model fitting may have also played a role in the resulting structural decomposition.

Reaching a firm conclusion on this matter would require making assessment of statistical significance of the model fit components identified with the stationary features and performing the SWD separation test for extreme SNR ratios. We defer this to future analysis of the data on 3C 120, while noting again that WISE has achieved its basic goal of providing an effective automated measure of kinematics in a jet with remarkably rapid structural changes.

The magnitude of the structural variability of the jet in 3C 120 is further emphasized in Fig. 16 showing the two-dimensional tracks of the SSP identified with WISE. The shape of individual tracks suggests a helical morphology, consistent with the patterns predicted from the modelling of the jet in 3C 120 with linearly growing Kelvin-Helmholtz instability (Hardee et al. 2005). In this framework, the observed evolution of the component tracks is consistent with the pattern motion of the helical surface mode of the instability identified in Hardee et al. (2005) to have a wavelength of ~ 3.1 jet radii and propagating at an apparent speed of ~ 0.8 c.

Hardee et al. (2005) also suggested that the structure of the flow is strongly dominated by the helical surface mode, which may explain the apparent lack of structural detail uncovered by
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Fig. 15. Core separation plot of the features identified in the jet of 3C 120. The model-fit based MOJAVE results (dashed lines) are compared to the WISE results (solid lines) obtained for the SWD scales 2 and 3 (selected in order to match the effective resolution of WISE to that of the Gaussian model fitting employed in the MOJAVE analysis).

Fig. 16. Two-dimensional tracks of SSP detected in 3C 120 at the scale 2 of SWD. The colored tracks correspond to the features plotted in Fig. 15 in the same color. The tracks are overplotted on a stacked-epoch image of the jet rotated by an angle of 0.4 radian. The plot confirms the presence of significant and evolving transverse structure in the jet, with individual tracks underlying the long-term evolution of the flow which becomes particularly prominent at core separations of $\gtrsim 6$ mas.

WISE on the finest wavelet scale. In this case, observations at a higher dynamic range would be needed to reveal the presence of higher (and weaker) modes of the instability developing in the jet on these spatial scales. Altogether the example of 3C 120 gives another demonstration of robustness of the WISE decomposition and analysis of a structural evolution that can be inferred from comparison of multiple images of an astronomical object.

7. Conclusions

The WISE method presented in this paper offers an effective and objective way for classifying structural patterns in images of astronomical objects and tracking their evolution traced by multiple observations of the same object. The method combines automatic segmented wavelet decomposition with multiscale cross...
correlation algorithm enabling reliable identification and tracking of statistically significant structural patterns to be performed. Tests of WISE performed on simulated images have demonstrated its capabilities for a robust decomposition and tracking of two-dimensional structures in astronomical images. Applications of WISE on the VLBI images of two prominent extragalactic jets have shown the robustness and fidelity of results obtained from WISE with those coming from the “standard” procedure of using multiple Gaussian components to represent the structure observed. The inherent multi-scale nature of WISE allows it also to go beyond the effective resolution of the Gaussian representation and to probe the two-dimensional distribution of structural displacements (hence probing the two-dimensional kinematic properties of the target object).

In addition to this, the multi-scale approach of WISE has several other specific advantages. Firstly, it allows for simultaneous detection of unresolved and marginally resolved features as well as extended structural patterns at low SNR. Secondly, the method provides a dynamic and structural scale-dependent account of the image noise, and uses it as an effective thresholding condition for assessing the statistical significance of individual structural patterns. Thirdly, multiple velocity components can also be distinguished by the method, if these components acting on different spatial scales – this can be a very important feature for studying the dynamics of optically thin emitting regions such as, for instance, stratified relativistic flows, with a combination of pattern and flow speed and strong transverse velocity gradients.

Combination of several scales also improves the cross-correlation employed by WISE, ensuring robust performance of the method in the case of severely undersampled data (with structural displacement between successive epochs becoming larger than the dimensions of the instrumental point spread function).

In its present realization, WISE performs well on structures with moderate extent, while it may face difficulties correctly identifying continuous structural details in which one of the dimensions is substantially smaller than the other (e.g., filamentary structure and thread-like features). If the ratio between the largest and smallest dimensions of such structure is smaller than the ratio of the maximum and minimum scales of WISE decomposition, the continuity of such structure may in principle be recognized. For more extreme cases, WISE will break the structure into two or more SSP which will be considered independent. A remedy to this deficiency may be found in considering groups of SSP during the MCC part of WISE, or in applying more generic approaches to feature identification (e.g., shapelets; cf. [Starck & Murtagh, 2006]).

Another issue requiring additional attention is the scale crossing of individual features that may occur as a result of expansion (as was illustrated by the example of 3C 273) or particular evolution of a complex three-dimensional emitting region projected onto the two-dimensional picture plane. At the moment, this issue has to be dealt with manually and outside of WISE, but an automated approach to this problem is clearly desired. One possibility here is to use the wavelet amplitudes associated with the same SSP at different scales, and to select the dominant scale adaptively based on the comparison of these amplitudes and their changes from one observing epoch to another.

Implementing this step may also require implementing robust error estimation for the locations, flux densities and dimensions of SSP identified by WISE. This can be done on the basis of SNR estimates performed at each individual scale of WISE decomposition. Generically, it is expected that and SSP detected with a given SNR at a particular wavelet scale l_w would have its positional and flux errors \( \propto l_w/\text{SNR}^2 \), while the error on the SSP dimension would be \( \propto l_w/\text{SNR}^{1/2} \) (cf. [Fomalont, 1999]). Such estimates can be implemented as a zeroth order approach, however a more detailed investigation of the errors estimates for the segmented wavelet decomposition is clearly needed.

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