Optical Distortion and its Representation

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Aims

- Due to off-axis reflective elements, there will be some degree of distortion (change) in pixel scale across the re-imaged focal planes.
- Distortion is significant. Code-V optical ray trace models: 4.7% (24μm), 7.9% (70μm - high resolution mode) and 9.7% (160μm). These numbers refer to the maximum fractional change in pixel scale.
- The requirement is to correct for distortion to better than 0.2 of a pixel on all arrays.
- No re-gridding is done to make the BCD and correct for distortion. Instead, the distortion will be represented in the header, inserted at the boresight-transfer stage.
- The mosaicer will read the distortion representation and re-grid the data to constant plate scale when making mosaics (BQD product).

Aims continued..

- How we plan to calibrate distortion for each FPA:
  - This will inevitably be a function of scan-mirror angle during an integration - although this has not yet been fully characterized.
  - The distortion will be represented by a polynomial. The coefficients will come out of the IOC Focal Plane Survey (D. Bayard and J. Keene, JPL).

- How we plan to represent distortion in FITS headers:
  - We will adopt the “TNX” representation for now which was initiated by IRAF. However, this is not a FITS standard.
  - This will be converted to a FITS-standard when ratified by the IAU. An example is the PV-system proposed by Calabretta & Greisen (2000), similar to that used in DSS images.
  - However, the IRAF group have debated this and proposed a more general form.
  - TNX will be used as a “stopgap” until a standard is ratified.

Background: The TNX Representation

- TNX is an “experimental” map projection recognized by the Image and Reduction Analysis Facility (IRAF), SAOimage, SAOtng, DS9, some “Starlink” packages and the World Coordinate System (WCS) libraries.

- Comprises a simple *Tangent sky projection (linear term) + Distortion (non-linear term)*. The distortion term depends on the instrument and accounts for non-linearity in plate scale, skew, asymmetries from all optical elements.
- To go from pixel coordinates (*x, y*) → sky coordinates in degrees (ξ, η):

1. Linear transform:

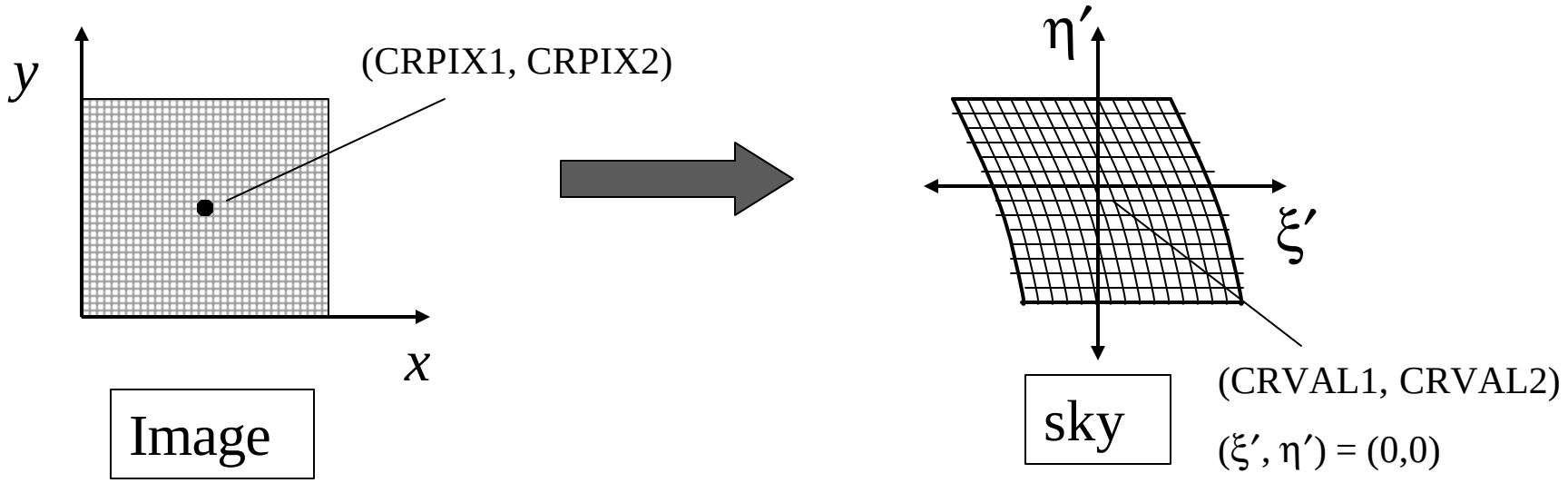
$$\begin{pmatrix} x \\ h \end{pmatrix} = \begin{pmatrix} CD_{11} & CD_{12} \\ CD_{21} & CD_{22} \end{pmatrix} \begin{pmatrix} x - \mathrm{CRPIX1} \\ y - \mathrm{CRPIX2} \end{pmatrix} \quad \text{(Pure TAN)}$$

2. Apply non-linear part:

$$\begin{pmatrix} x' \\ h' \end{pmatrix} = \begin{pmatrix} x \\ h \end{pmatrix} + \begin{pmatrix} \mathrm{lngcor}[x,h] \\ \mathrm{latcor}[x,h] \end{pmatrix}$$

Distortion, polynomial fit

TNX Representation continued..



CRPIX1, CRPIX2 = pixel coordinates of tangent point

CRVAL1, CRVAL2 = *RA*, *DEC* of tangent point on sky

To compute physical world coordinates (*RA*, *DEC*) of any point in the image:

$RA = \mathrm{CRVAL1} + \xi'$

$DEC = \mathrm{CRVAL2} + \eta'$

TNX FITS Header Keywords

- The coordinates of the tangent point: CRVAL and CRPIX keywords.

- Coordinate transformation matrix elements (CD_{ij}). These replace the old CROTA2 and CDELT1, CDELT2 keywords associated with a pure TAN projection - the rotation and image scale keywords:

$$\begin{pmatrix} CD_{11} & CD_{12} \\ CD_{21} & CD_{22} \end{pmatrix} \equiv \begin{pmatrix} \mathrm{CDELT1}\cos(\mathrm{CROTA2}) & -\mathrm{CDELT2}\sin(\mathrm{CROTA2}) \\ \mathrm{CDELT1}\sin(\mathrm{CROTA2}) & \mathrm{CDELT2}\cos(\mathrm{CROTA2}) \end{pmatrix}$$

- The WAT (World-Attribute) keywords which define the order, cross-terms and coefficients of the polynomials associated with the distortion terms lngcor(ξ, η) and latcor(ξ, η) above. These are expressed in terms of a polynomial in ξ and η. For a cubic polynomial with 10 coefficients:

$$\mathrm{lngcor}(x,h) = \mathrm{c}_1 + \mathrm{c}_2 x + \mathrm{c}_3 x^2 + c_4 x^3 + c_5 h + c_6 x h + c_7 x^2 h + c_8 h^2 + c_9 h^2 x + c_{10} h^3,$$

the header will look like:

Example Header

- TNX keywords generated by *ccmap* task in *IRAF* for cerro-tololo 4m (numbers in bold face are the 10 polynomial coefficients).

```
CTYPE1  = 'RA---TNX'           / Coordinate type
CTYPE2  = 'DEC--TNX'           / Coordinate type
CRVAL1  =   310.08145293602507 / Coordinate reference value
CRVAL2  =   20.663666538998399 / Coordinate reference value
CRPIX1  =            68.3258 / Coordinate reference pixel
CRPIX2  =            67.2481 / Coordinate reference pixel

CD1_1   =       -6.8295807e-08 / Coordinate matrix
CD2_1   =        7.3313414e-05 / Coordinate matrix
CD1_2   =         7.374228e-05 / Coordinate matrix
CD2_2   =       -1.1927219e-06 / Coordinate matrix

WAT0_001= 'system=image'       / Coordinate system
WAT1_001= 'wtype=tnx axtype=ra lngcor = "3. 4. 4. 2. -0.3171856965643079 -0.015'
WAT1_002= '0652479325533 -0.3126038394350166 -0.1511955040928311 0.002318100364
WAT1_003= '838772 0.0174913420424022 -0.0108274423020123 -0.138796673564234
WAT1_004= '-4.30730976939804E-4 0.00906928008295441 0.00287526278754504 -0.0
WAT1_005= '4487658756007625 -0.1058043162287004 -0.0686214765375767 "

WAT2_001= 'wtype=tnx axtype=dec latcor = "3. 4. 4. 2. -0.3171856965643079 -0.01'
WAT2_002= '50652479325533 -0.3126038394350166 -0.1511955040928311 0.00553481957'
WAT2_003= '8784082 0.0125890793029932 0.0101678085575339 0.0154183298696018'
WAT2_004= '0.0353197958941362 0.015009645430599 -0.108647952595234 0.0399806'
WAT2_005= '086902122 0.02341002785565408 -0.07773808393244387 "
```

Distortion in RA

Distortion in DEC

Conversion to the PV system

- Most of the functionality of TNX (the CD and WAT keywords) will be absorbed by a more general FITS TAN projection. One proposal is the PV system (see Calabretta & Greisen, 2000).

- There is a restriction on the format of the TNX projection parameters so it can be easily mapped into the PV system (From Lindsey Davis, NRAO):

  1. The function must be a pure polynomial (no legendre or chebyshev).

  2. The order of this polynomial in both ξ and η must be ≤ 8 with half-cross terms (22 coefficients), or, ≤ 4 for full-cross terms (16 coefficients).

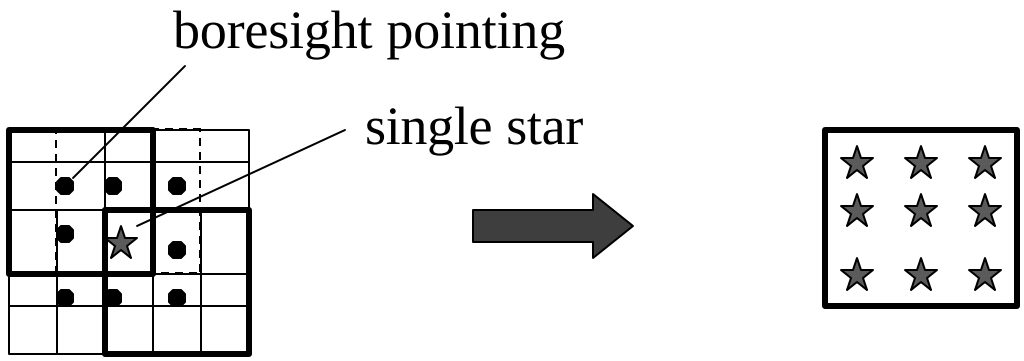
- The sky coordinates can then be expressed in terms of new coefficients which are all FITS keywords: PVi_1, PVi_2, PVi_3 . . . ; PVj_1, PVj_2, PVj_3 . . . , where

$$x' = PVi_1 + PVi_2 x + PVi_{31} h + PVi_4 x^2 + PVi_5 xh + PVi_6 h^2 + \ldots$$

$$h' = PVj_1 + PVj_2 x + PVj_{31} h + PVj_4 x^2 + PVj_5 xh + PVj_6 h^6 + \ldots$$

Calibrating the Optical Distortion?

- Current strategy for IOC consists of acquiring images of a single bright star at 9 uniformly spaced positions on the array spanning the complete range of mirror angles (D. Bayard and J. Keene, JPL). There are no bright enough clusters to do this robustly.
- Since IRAF is the only package that generates TNX keywords, use the above survey information to compute a set of keywords directly *for each* scan-mirror angle.
- The IRAF task "*ccmap*" can be used to read in a table of extracted *x,y* positions from an image and corresponding *RA, DEC* (effectively distortion-free tangent points) provided by the star-tracker offsets and compute all desired polynomial coefficients.



Keywords in Science Data

- Since the distortion (TNX polynomial coefficients) will be calibrated as a function of scan-mirror angle, the distortion for arbitrary mirror angles will be computed by linearly interpolating each coefficient.

- Is interpolation necessary? The distortion could have very little dependence on scan-mirror angle.

- These new coefficients will then be inserted into FITS headers and tested to ensure that known sources map correctly into their sky coordinates.