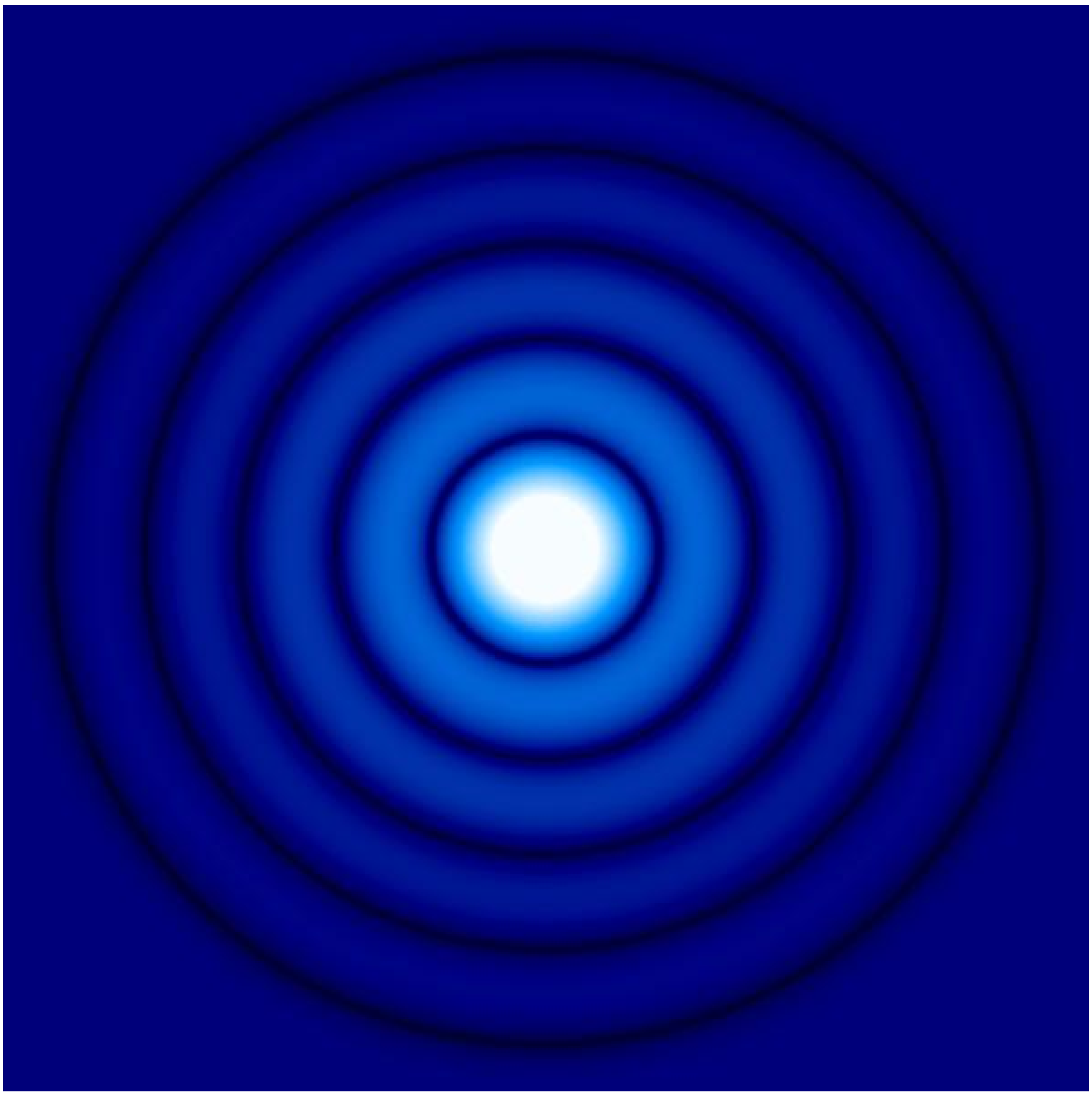
POINT SPREAD FUNCTIONS



"Point Spread Functions" describe the two-dimensional distribution of light in the telescope focal plane for astronomical point sources. Modern optical designers put a great deal of effort into reducing the size of the PSF for large telescopes.

Good PSF evaluation is especially critical for telescopes which are intended to have near-diffraction limited performance. That obviously includes space telescopes. But it also includes large ground-based telescopes which are equipped with "active" or "adaptive" optics sytems, which can greatly reduce the effects of atmospheric seeing on the PSF.

The PSF for a perfect optical system, based on circular elements, would be an "Airy Pattern," which is derived from Fraunhofer diffraction theory (scalar approximation applied to plane waves).

The Airy pattern is given by the following expression:

$$I(u) = \frac{1}{(1-\epsilon^2)^2}\left[\frac{2\,J_1(u)}{u} - \epsilon^2\,\frac{2J_1(\epsilon u)}{\epsilon u}\right]^2$$

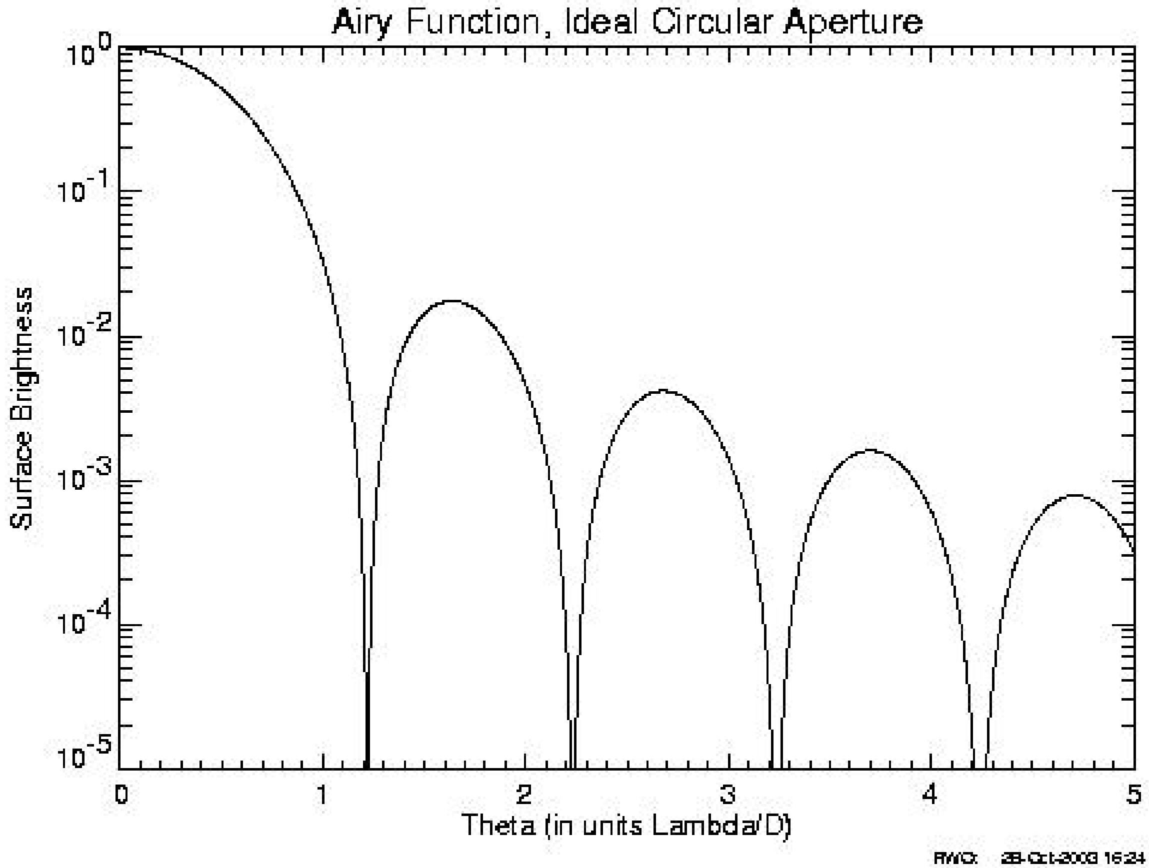
$I(u)$ **is the surface brightness in the focal plane, normalized to its maximum at** $u = 0$

u **is a dimensionless distance from the optical axis in the focal plane and is related to the angular radius** θ **(as measured from the primary aperture) and the diameter** D **of the primary aperture as follows:**

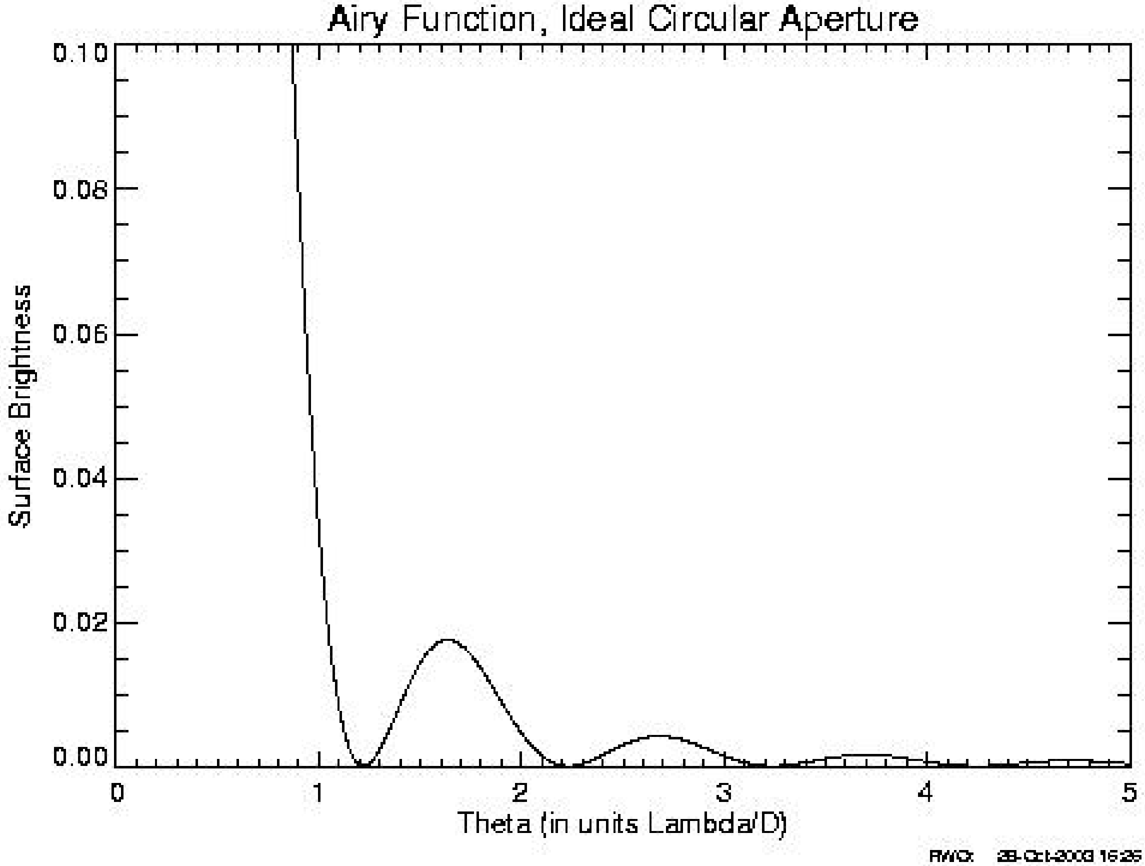
$$u = \frac{\pi}{\lambda}\, D\,\theta$$

ϵ **is the fractional radius of the central obscuration of the primary aperture (assumed circular). Can be 0.**

J_1 **is the order 1 Bessel function of the first kind.**



Airy Pattern (zero obscuration) in log intensity space showing the structure of the ring pattern.



Airy Pattern (zero obscuration) in linear intensity space (enlarged y axis)

THE AIRY PATTERN

The Airy pattern is shown as an image on the first page and is plotted on the two preceding pages.

It consists of a series of sharp dark rings alternating with broader bright rings.

The pattern drops to 50% power at a radius of 0.514 λ/D, so its $FWHM = 1.028\lambda/D$.

The core of the Airy Pattern, inside the first minimum is often called the "Airy Disk." It contains some 86% of the total light in the image.

The first three dark rings occur at radii of 1.22, 2.23, and 3.24 λ/D, respectively.

ENCIRCLED ENERGY

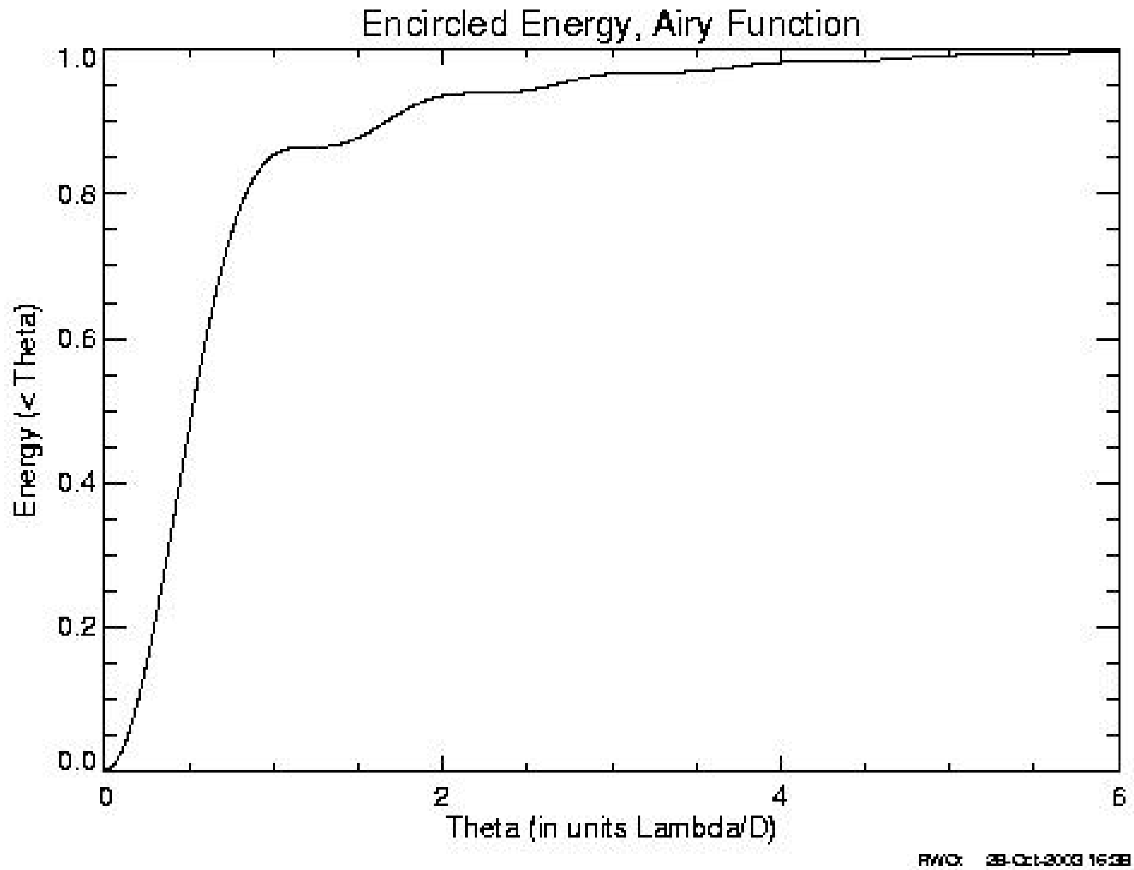
The size of the pattern is important for purposes of resolving sources, but for photometric measurements, we are more interested in the encircled energy distribution of the pattern. The encircled energy is the fraction of the total integrated flux in the image contained within a given radius r.

That is, $EE(R) = \int_0^R 2\,\pi\, I(r)\, r\, dr / E_0$,

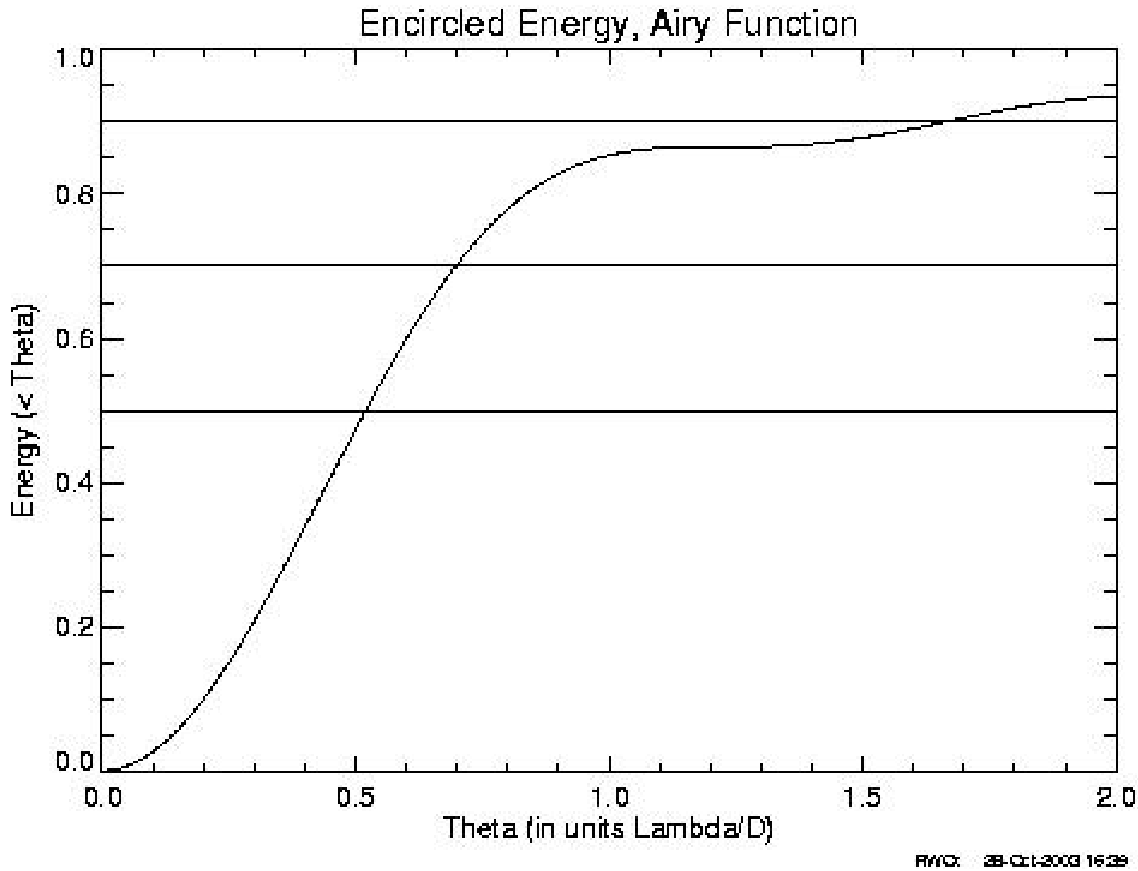
where $E_0 = \int_0^\infty 2\,\pi\, I(r)\, r\, dr$

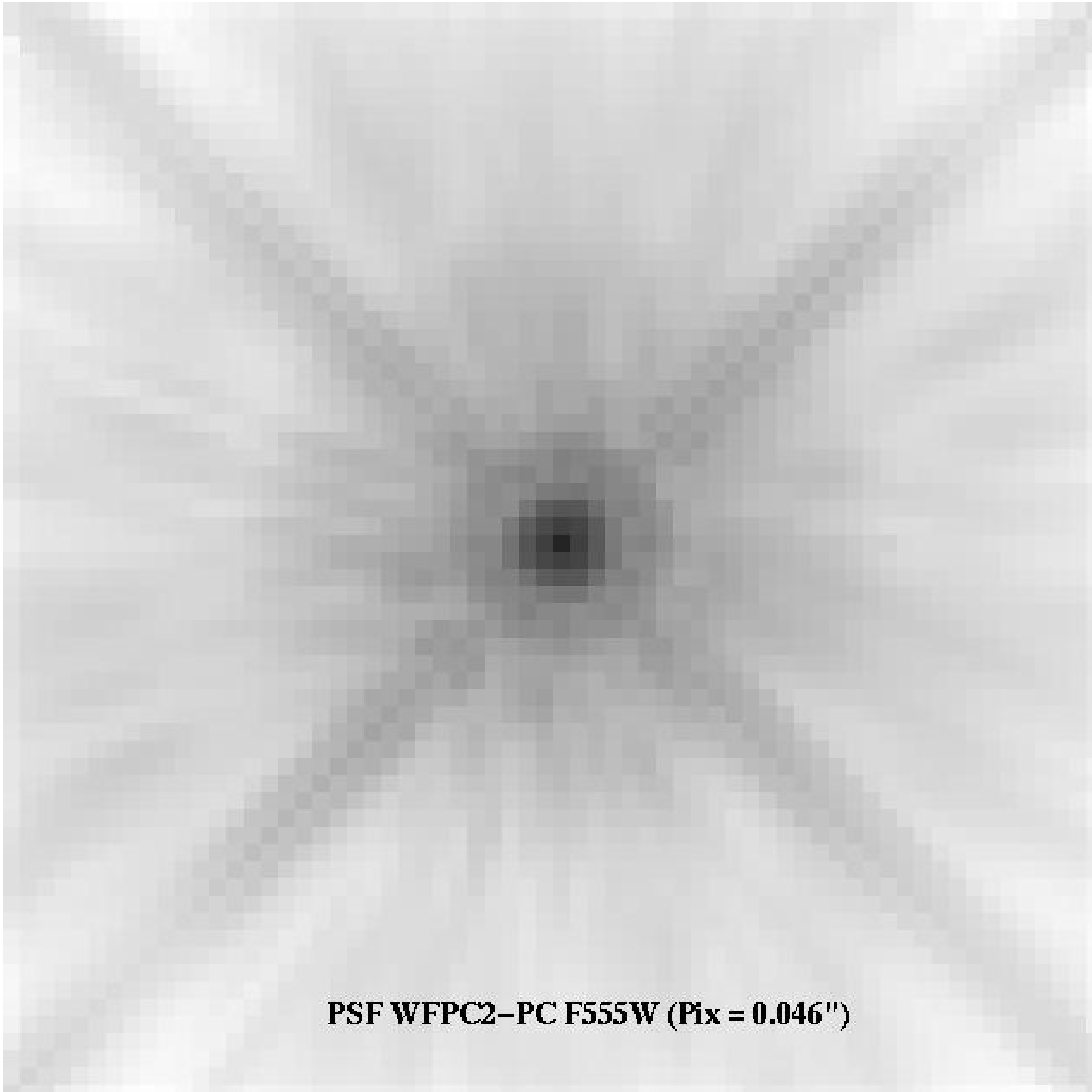
Real PSF's often have very extended wings, so for the purpose of characterizing photometric performance we often use the 70% encircled energy diameter, EED_70.

The plots on the next pages show the EE function for the Airy pattern.

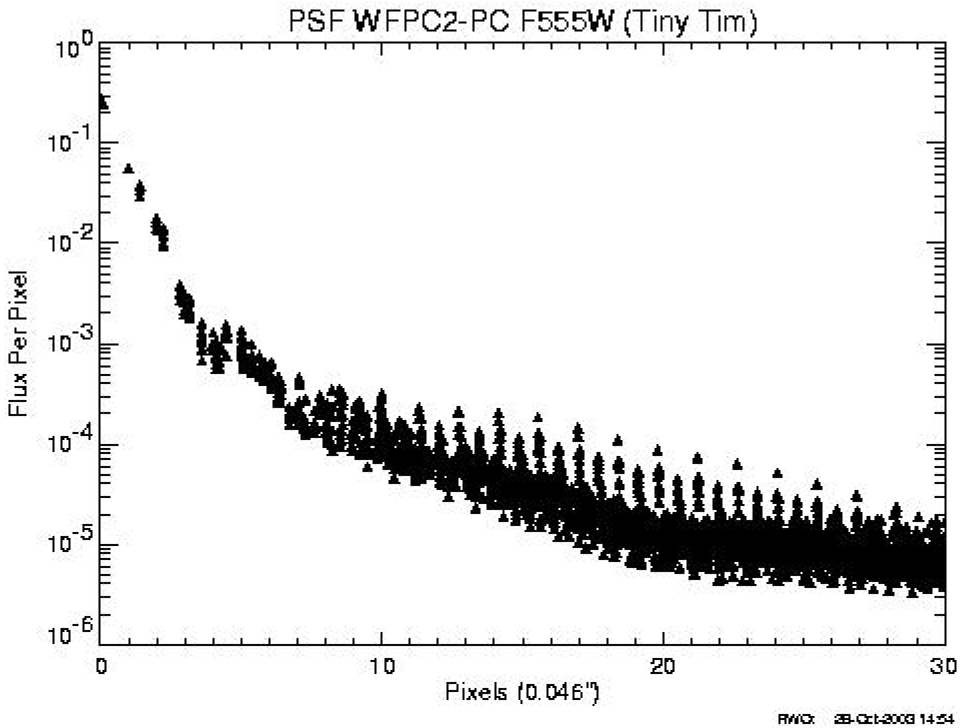


θ (λ/D units)	$EE(\theta)$
0.52	0.50
0.70	0.70
1.22	0.86
1.66	0.90

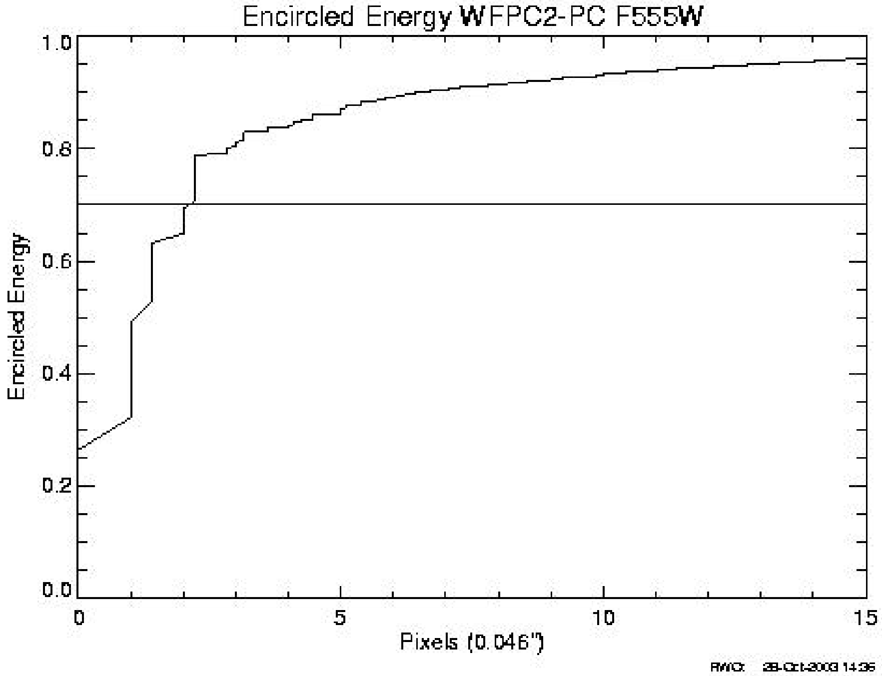

Encircled Energy, Airy Function
Energy (< Theta)
Theta (in units Lambda/D)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0



Point spread function image for the HST/WFPC-2 PC chip (Tiny Tim simulation)



Azimuthally averaged profile of WFPC2-PC PSF



Encircled energy for the WFPC2-PC PSF. The 70% EE radius is 2 pixels or 0.092″