Analytic Solution for Fowler-Sampling Linearity Correction

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We assume that the IRAC non-linearity model is a quadratic function. The non-linear DN are given by the quadratic function:

$$D' = \alpha D^2 + D \tag{1}$$

where D is the linear DN. The linear DN can be described as a rate (R) times time (t),

$$D = Rt \tag{2}$$

so

$$D' = \alpha R^2 t^2 + Rt \tag{3}$$

assume for simplicity that that R is in units of DN per fowler time sample length, so that t can be expressed in IRAC read times (i),

$$t = i \tag{4}$$

The exact definition of the fowler sampled result is:

$$D' = -\frac{1}{n}\sum_{1}^{n}\left(\alpha R^2 i^2 + Ri\right) + \frac{1}{n}\sum_{w+n}^{w+2n}\left(\alpha R^2 i^2 + Ri\right) \tag{5}$$

where n is the fowler number and w is the number of wait periods. The left summa is the set of pedestal reads, and the second summa is the set of signal reads. Included here is the normalization by fowler number, which will normally be done on-board as a result of the barrel-shifting. Rearranging, this is

$$D' = \alpha R^2 \left[\frac{1}{n}\left(-\sum_{1}^{n} i^2 + \sum_{w+n}^{w+2n} i^2\right)\right] + R\left[\frac{1}{n}\left(-\sum_{1}^{n} i + \sum_{w+n}^{w+2n} i\right)\right] \tag{6}$$

The terms in brackets are constants for any given value of w and n. Define the left-hand term in brackets to be A and the right-hand term to be B. Thus

$$D' = \alpha A R^2 + B R \tag{7}$$

The solution for the rate R is then

$$R = \frac{-B + \sqrt[2]{B^2 - 4A\alpha D'}}{2A\alpha} \tag{8}$$

but we don't want the rate, we want D, the linear DN. The linear DN is given by the rate R times the exposure time (w+n). Thus

$$D = R(w+n) = (w+n)\frac{-B + \sqrt[2]{B^2 - 4A\alpha D'}}{2A\alpha} \tag{9}$$

Furthermore, evaluating B we find that

$$B = \frac{1}{n}\left[-\sum_{1}^{n} i + \sum_{w+n}^{w+2n} i\right] = (w+n) \tag{10}$$

Using Frank Masci's terminology from SLOPECORR, substituting (10) into (9) we get

$$D = \frac{-1 + \sqrt[2]{1 - 4L\alpha D'}}{2L\alpha} \tag{11}$$

where

$$L = \frac{A}{B^2} = \frac{1}{n(w+n)^2}\left[-\sum_{1}^{n} i^2 + \sum_{w+n}^{w+2n} i^2\right] \tag{12}$$

The validity of the above equation has been verified via an IRAC simulator. A simulator was written which computed the observed DN by duplicating the fowler sampling scheme using an observed flux rate and the quadratic non-linearity. The simulated, observed non-linear DN were then inverted using the modified quadratic inversion in equations 11 and 12. This correctly reproduced the real, non-linear DN.

Incidentally, technically equation 4 should be

$$t = ci \tag{13}$$

where c equals 0.2 seconds for full-array and 0.01 seconds for subarray. However, this makes equation 6

$$D' = \alpha R^2 \left[\frac{c^2}{n}\left(-\sum_{1}^{n} i^2 + \sum_{w+n}^{w+2n} i^2\right)\right] + R\left[\frac{c}{n}\left(-\sum_{1}^{n} i + \sum_{w+n}^{w+2n} i\right)\right] \tag{14}$$

when the ratio A/B^2 is taken, the constant c cancels.